Synthetic Seismograms in Heterogeneous Media and Study of Formation and Propagation of Regional Phases

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Contract No. F49620-95-1-0028

Abstract

The object of this project is aimed at development and application of a new 3D wave propagation and modeling method in complex heterogeneous media using one-way wave approximation to study many outstanding problems of regional wave propagation in the context of nuclear test monitoring. The great advantages of one-way propagation methods are the fast speed of computation, often by several orders of magnitudes faster than finite difference and finite element methods, and the huge saving in internal memory. The first year's effort is concentrated on developing theory and techniques for fast calculation of reflected or backscattered waves, namely methods of synthetic seismograms using the concept of one-way wave propagation and single interaction. The research effort supported by this contract has produced the following results regarding the backscattered field modeling. Wu and Huang (1995a, b) has introduced a wide-angle modeling method for acoustic waves using the De Wolf approximation and phase-screen propagator. Xie and Wu (1995) introduced a complex screen method for the calculation of backscattered elastic waves under small-angle approximation. Wu (1995) extended the wide-angle method to the case of elastic waves, and derived the relation between the wide-angle method and the screen approximation for the acoustic case. Numerical examples showed excellent results of the method. Comparison of angular dependence of reflection coefficients calculated from synthetic seismograms by this method with the theoretical curves showed good agreement between theory and synthetics. The wide-angle version of the method can even model the critical and post-critical reflections. From the comparison of synthetic seismograms made by this fast method and by finite difference calculations the validy and applicability of the method has been demonstrated. Even the screen approximation method which involves a small-angle approximation for the medium-wave interaction, can produce satisfactory synthetic seismograms for many practical cases. The wide-angle version of the method, though more computationally intensive than the screen method, will have a wide range of applications. The next year's effort will be concentrated on solving the free surface and Moho reflections and applying the method to the synthetic seismograms of regional phases.

1 Objective

The new task for monitoring a Comprehensive Test Ban Treaty and the current Nuclear Non-Proliferation Treaty presents a great challenge to the existing methods using regional phases. This is because the nature and properties of regional phases remain unclear due to the lack of theory and efficient methods for computing synthetic seismograms in complex environments. The object of this project is aimed at development and application of a new 3D wave propagation and modeling method in complex heterogeneous media using one-way wave approximation to study many outstanding problems of regional wave propagation in the context of nuclear test monitoring. The great advantages of one-way propagation methods are the fast speed of computation, often by several orders of magnitudes faster than finite difference and finite element methods, and the huge saving in internal memory. The new project will take the advantages of the recent progress in elastic one-way wave propagation theory (Wu, 1994) and continue to develop the theory and method for the purpose of regional wave synthetic seismograms. The major effort for the new project will be to solve the problems of backscattering calculation and free-surface/Moho reflections so that the one-way wave theory and method can be applied to the regional waveguide environment. Then the influence of upper and lower crustal small-scale heterogeneities to Lg propagation and attenuation, and other path effects will be examined by numerical simulations using the new method. The development of the theoretical model will be combined with analysis of the Eurasian regional phase observations to extract quantitative information from regional phases.

2 Research Accomplished

The recent successful extension and applications of one-way elastic wave propagation methods, e.g. the complex screen method (Wu, 1994; Wu and Xie, 1994), stimulated the research interest in developing similar theory and techniques for reflected or backscattered wave calculation, namely methods of synthetic seismograms using the concept of one-way wave propagation and single interaction. The research effort supported by this contract has produced the following results regarding the backscattered field modeling. Wu and Huang (1995a, b) has introduced a wide-angle modeling method for acoustic waves using the De Wolf approximation and phase-screen propagator. Xie and Wu (1995) introduced a complex screen method for the calculation of backscattered elastic waves under small-angle approximation. Wu (1995) extended the wide-angle method to the case of elastic waves, and derived the relation between the wide-angle method and the screen approximation for the acoustic case. The validity of the method and the wide-angle capability for the dual-domain implementation are demonstrated by numerical examples.

2.1 Synthetic Seismograms in Heterogeneous Media By One-Return Approximation

A new method based on multiple-forescattering single-backscattering (MFSB) approximation, i.e. the one-return approximation (the De Wolf approximation) for calculating backscattered fields was introduced (Wu and Huang, 1995a, b; Wu, 1995). When discontinuities inside a medium are not very sharp or parameter perturbations of heterogeneities are not very strong, reverberations between heterogeneities or resonance scattering may be neglected. However, accumulated effect of forward scattering usually can not be neglected. In such cases, the Born approximation is not valid but the De Wolf approximation can be applied.

The Lipmann-Schwinger equation has a formal solution of multiple scattering Born series. The widely used Born approximation is the leading term of the series. After renormalization of the multiple scattering series, De Wolf (1971, 1985) derived a MFSB approximation given by

$$p(\mathbf{x}) = p^f(\mathbf{x}) + k^2 \int_v d^3\mathbf{x}' g^f(\mathbf{x};\mathbf{x}') F(\mathbf{x}') p^f(\mathbf{x}') \,, \tag{1}$$

where p^f and g^f are the renormalized, multiple forescattered field and Green's function, respectively, F is the perturbation function or equivalent force which is a scalar function in the case of scalar media, but involves vector operators, such as gradient and divergence operators, and tensor operators, such as the strain operators and double divergence operator, in the case of acoustic and elastic media, respectively. The renormalized p^f and g^f will be calculated using the phase-screen or complex-screen propagator.

To speed up the calculation of backscattered fields, the local Born approximation can be used within a thin-slab (*cf.* Fig. 1). This means that the forescattered field p^f can be kept unperturbed and g^f can be replaced by a constant medium Green's function within the slab. Assume z' and z_1 as the slab entrance (top) and exit (bottom) respectively (*cf.* Fig. 1), and Fourier-transform above equation with respect to $\mathbf{x}_T$, we get a dual-domain expression for the calculation of scattered fields. In the case of scalar media, the dual-domain expression is

$$P(\mathbf{K}_T, z^*) = \frac{i}{2\gamma} k^2 \int_{z'}^{z_1} dz e^{i\gamma|z^*-z|} \iint d^2\mathbf{x}_T e^{-i\mathbf{K}_T\cdot\mathbf{x}_T} [F(\mathbf{x}_T, z) p^f(\mathbf{x}_T, z)] \,. \tag{2}$$

where z^* is the receiver depth, x_T is the horizontal coordinates, K_T is the horizontal spatial frequency, and $\gamma = \sqrt{k^2 - K_T^2}$. Note that the two dimensional inner integral is a 2-D Fourier transform. Therefore, the dual-domain technique can be used to implement the equation.

For the case of acoustic media, the dual-domain formulation is

$$\begin{aligned} P(\mathbf{K}_T, z^*) = \frac{i}{2\gamma} k^2 \int_{z'}^{z_1} dz e^{i\gamma|z^*-z|} \Big\{ &\int d^2\mathbf{x}_T e^{-i\mathbf{K}_T\cdot\mathbf{x}_T} [\varepsilon_\kappa(\mathbf{x}_T, z) p^f(\mathbf{x}_T, z)] \\ &+ \frac{i}{k}\hat{k} \cdot \int d^2\mathbf{x}_T e^{-i\mathbf{K}_T\cdot\mathbf{x}_T} [\varepsilon_\rho(\mathbf{x}_T, z) \nabla p^f(\mathbf{x}_T, z)] \Big\} \,, \end{aligned} \tag{3}$$

where $\varepsilon_\kappa(\mathbf{x}) = \kappa_0/\kappa(\mathbf{x}) - 1$, $\varepsilon_\rho(\mathbf{x}) = \rho_0/\rho(\mathbf{x}) - 1$, and $\hat{k} = \frac{1}{k}(\mathbf{K}_T, k_z)$, with $k_z = \pm\gamma$ for forescattering and backscattering, respectively. When the receiving level is at the bottom of the thin-slab (forescattering), $z^* = z_1$;while $z^* = z'$ is for the backscattered field at the top of the thin-slab. The total transmitted filed at the slab bottom can be calculated as the sum of the forescattered field and the primary field.

The dual-domain expressions for the case of elastic media is more complicated and involved tensor operations. Readers are referred to Wu (1995).

2.2 The screen approximation

For some special applications, the synthetics only involve small-angle backscattering. In this case the screen approximation can be applied to accelerate the computation.

Under small-angle scattering approximation, we can compress the thin-slab into an equivalent screen and therefore change the 3-D spectrum into a 2-D spectrum. Dual-domain implementation of the screen approximation will make the modeling of backscattering very efficient. In the following the case of acoustic media will be shown as an example. For the screen approximation in the case of elastic media, readers are referred to Xie and Wu (1995) and the forthcoming publications.

With the screen approximation, scattered field can be calculated as

$$P(\mathbf{K}_T, z^*) \approx i\frac{k^2}{2\gamma} e^{ik_z(z^*-z')} \iint d\mathbf{x}_T e^{-i\mathbf{K}_T\cdot\mathbf{x}_T} S(\mathbf{x}_T) p^0(\mathbf{x}_T) \tag{4}$$

where

$$S(\mathbf{x}_T) = S_V(\mathbf{x}_T) = \int_0^{\Delta z} dz[\varepsilon_\kappa(\mathbf{x}_T, z) - \varepsilon_\rho(\mathbf{x}_T, z)] \;\; for\; forescattering \tag{5}$$

is the velocity screen and

$$S(\mathbf{x}_T) = S_I(\mathbf{x}_T) = \int_0^{\Delta z} dz e^{i2kz}[\varepsilon_\kappa(\mathbf{x}_T, z) + \varepsilon_\rho(\mathbf{x}_T, z)] \;\; for\; backscattering \tag{6}$$

is the impedance screen.

2.3 Numerical examples

First we compare the calculated reflection coefficients by the one-return method with the theoretical ones for plan wave incidences. The model is shown in Fig. 2, where a plane interface between two layers is located at the depth of 500m. The model is defined on a 2048×300 rectangular grid. The grid spacing in the horizontal direction is 8m and that in the vertical direction is 5m. A pressure point source is applied at the center of the upper border of the model. The velocity and density of the upper layer are 2000m/s and 1.0 g/cm^3, respectively. The frequency range used in the calculation is from 14.6Hz to 19.5Hz with 11 frequency components. We calculate the reflection coefficient for each frequency component and take an average over the 11 frequencies.

Fig. 3a shows the results for 10% of velocity and density perturbations in the lower layer and Fig. 3b is for 20%. In both figures, the dashed lines are theoretically predicted reflection coefficients of plane wave incidence and the solid curves are calculated results. We can see the good agreement between the theoretical ones and the synthetics when the incidence angles are smaller than the critical angles. When the incidence angles are near and beyond the critical angles, the numerical results deviate from the theoretical curves. This may be due to several reasons such as the curved wavefronts of waves from the point source, the wavenumber filtering in the process of forward propagation which reduces the amplitudes of large-angle scattered waves, and the effect of finite layer thickness. When the velocity and density perturbations of the lower layer are -10% and -20%, the corresponding results are given in Fig. 4a and 4b, respectively. We see from these figures that the synthetic reflection coefficients agree well with the theoretical results when the incidence angles are smaller than approximately 70°. For larger incidence angles, the amplitudes of reflection coefficients decrease because of the wavenumber filtering.

The next two examples is for the test of screen approximation for synthetic seismograms. Fig. 5 shows the model space for scalar wave modeling. A high velocity cylinder with 10% perturbation is situated at the middle of the model space. A plane wave is incident vertically down to the model space. The source time function is a Gaussian first derivative with the dominant frequency of $30Hz$. The screen interval is 10 m inside the cylinder; while the horizontal grid spacing is 3.125m. Fig. 6a shows the comparison of synthetic seismograms calculated by the screen approximation and by the finite difference method. In the figure only the transversal components are shown. We see good agreement between the results of finite difference calculations and this method.

The model for the elastic wave scattering is similar to that in Fig. 5. The parameters for the background media are $\alpha = 3500m/s$, $\beta = 2050m/s$ and $\rho = 2200kg/m^3$. The inclusions have a 5% perturbation for both P- and S-wave velocities. The diameter of the elastic cylinder is 300m. A plane P-wave is incident on the cylinder. Fig. 6b gives the synthetic seismograms from a receiver array 700m away from the center of the cylinder. The synthetics marked with "SCREEN" is from the complex-screen method while with "FD", from the finite-difference method. The upper panel is for $x-$component (transverse component) and the lower panel is for $z-$component (longitudinal component). Shown in the figure are two P-arrivals and two S-arrivals which are reflections from both the upper and lower boundaries of the cylinder. The results show general agreements in both amplitude and arrival time. Since the receiver array is only 550m from the border of the cylinder, the receiving profile spans a rather wide range of scattering angle. That means that even the small angle approximation can give satisfactory results in many cases.

3 Conclusions and Recommendations

We have made significant progress in modeling backscattering. Synthetic seismograms showed good agreement with finite difference calculations in our numerical examples. Next year's effort will be concentrated on solving the problems of free surface and Moho reflections and applying the method to simulate regional wave propagations

References

[1] De Wolf, D.A., 1971. Electromagnetic reflection from an extended turbulent medium: cumulative forward-scatter single-backscatter approximation, *IEEE trans. Ant. and Propag.*, **AP-19**, 254-262.

[2] De Wolf, D.A., 1985. Renormalization of EM fields in application to large-angle scattering from randomly continuous media and sparse particle distributions, *IEEE trans. Ant. and Propag.*, **AP-33**, 608-615.

[3] Wu, R.S., 1994, Wide-angle elastic wave one-way propagation in heterogeneous media and an elastic wave complex-screen method, *J. Geophys. Res., 99,* 751-766.

[4] Wu, R.S., 1995, Synthetic seismograms in heterogeneous media by one-return approximation, submitted to Phys. Earth & Planet. Inter..

[5] Wu, R.S. and Huang, L.J., 1992. Scattered field calculation in heterogeneous media using phase-screen propagator. Expanded Abstracts of the Technical Program, SEG 62th Annual Meeting: 1289-1292.

[6] Wu, R.S. and Huang, L.J., 1995. Reflected wave modeling in heterogeneous acoustic media using the De Wolf approximation. to be appeared in "Mathematical methods in geophysical imaging III".

[7] Wu, R.S., Huang, L.J. and X.B. Xie, 1995. Backscattered wave calculation using the De Wolf approximation and a phase-screen propagator. submitted to Expanded Abstracts of the Technical Program, SEG 65th Annual Meeting.

[8] Wu, R.S. and X.B. Xie, 1994. Multi-screen backpropagator for fast 3D elastic prestack migration. Mathematical Methods in Geophysical Imaging II, SPIE vol. 2301: 181-193.

[9] Xie, X.B. and Wu, R.S., 1995. A Complex-Screen Method for Modeling Elastic Wave Reflections. submitted to Expanded Abstracts of the Technical Program, SEG 65th Annual Meeting.

Figure 1: Geometry of the thin-slab formulation.

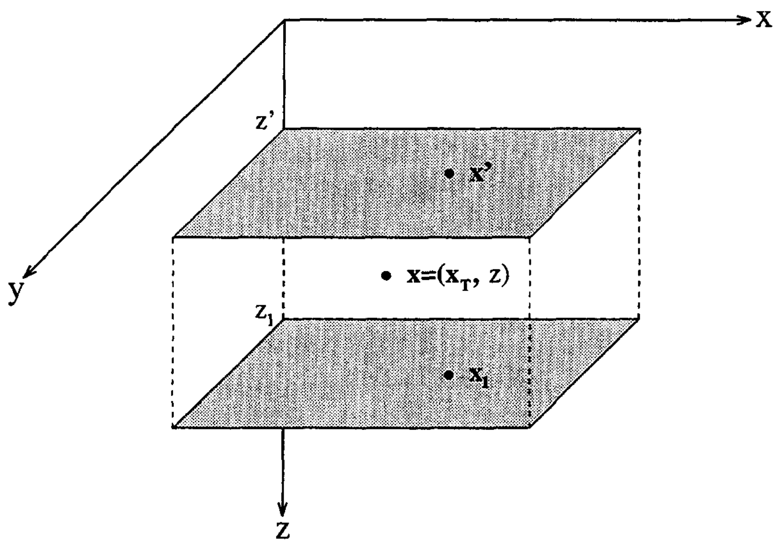
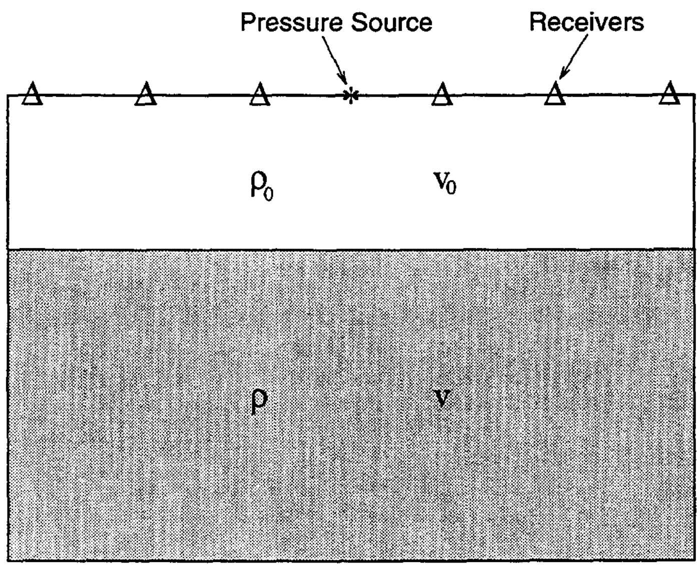
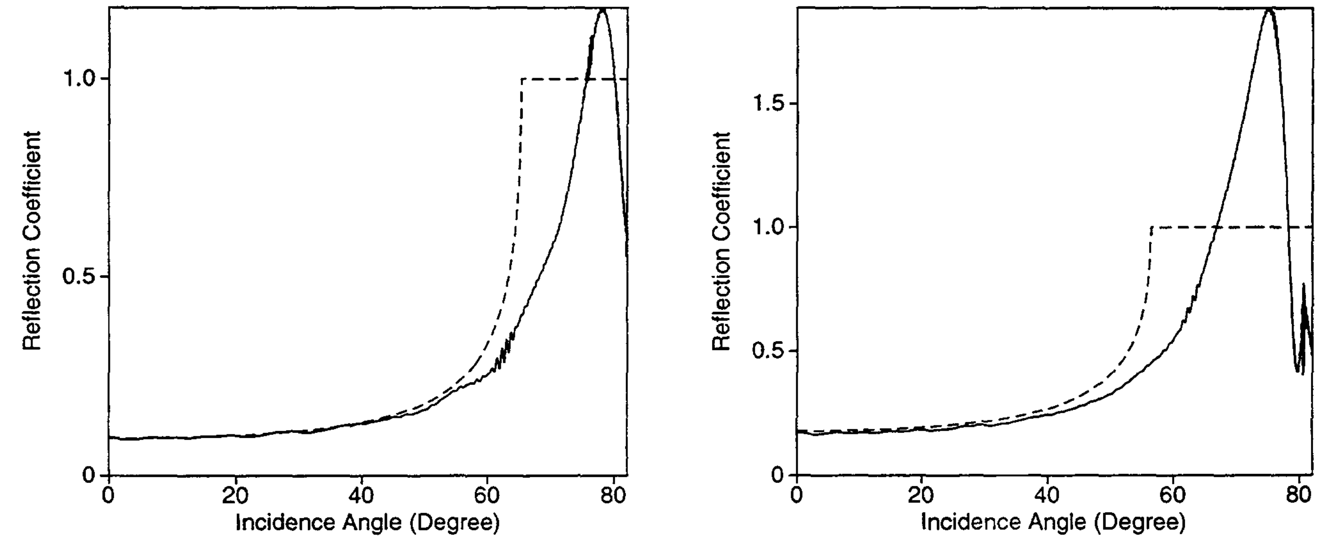


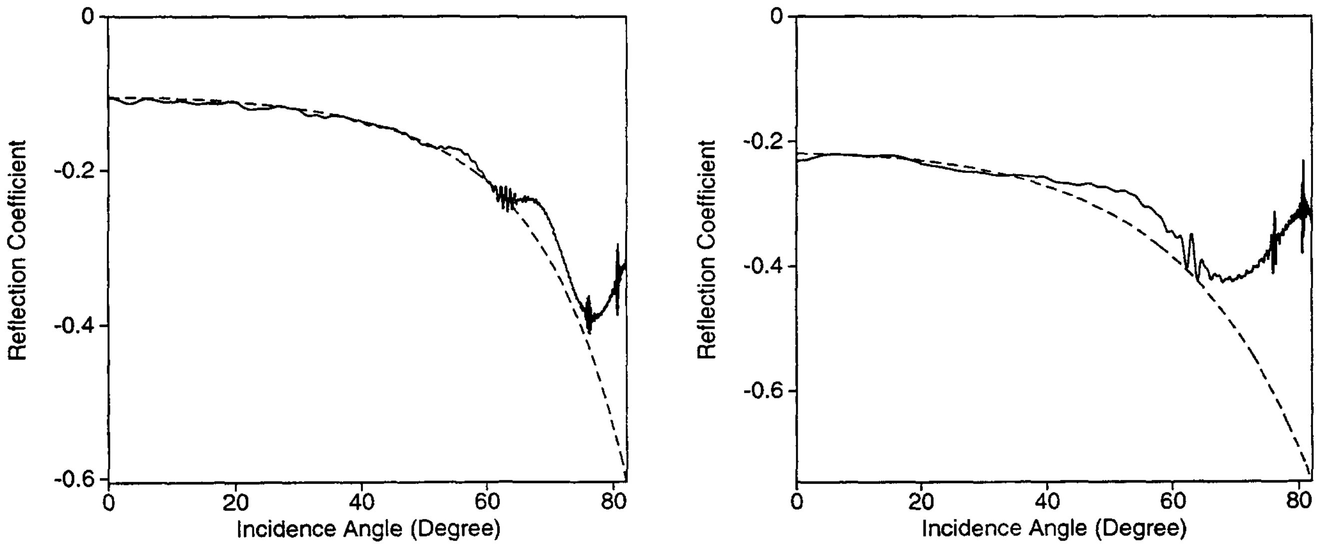
Figure 2: The layered model used in the numerical simulation.





(a) (b)

Figure 3: Comparison between theoretical and simulated reflection coefficients for a high-velocity 'half-space' shown in Figure 2 when the velocity and density perturbations of the lower 'half-space' are (a) 10% and (b) 20%. Dashed lines represent the theoretical reflection coefficients and the solid curves, the simulated results.



(a) (b)

Figure 4: Comparison between theoretical and simulated reflection coefficients for a low-velocity 'half-space' shown in Figure 2 when the velocity and density perturbations of the lower 'half-space' are (a) -10% and (b) -20%. Dashed lines represent the theoretical reflection coefficients and the solid curves, the simulated results.

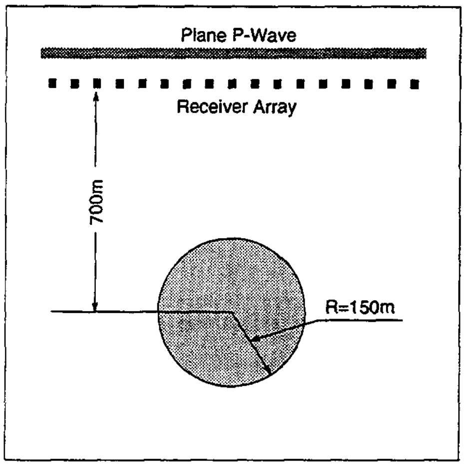
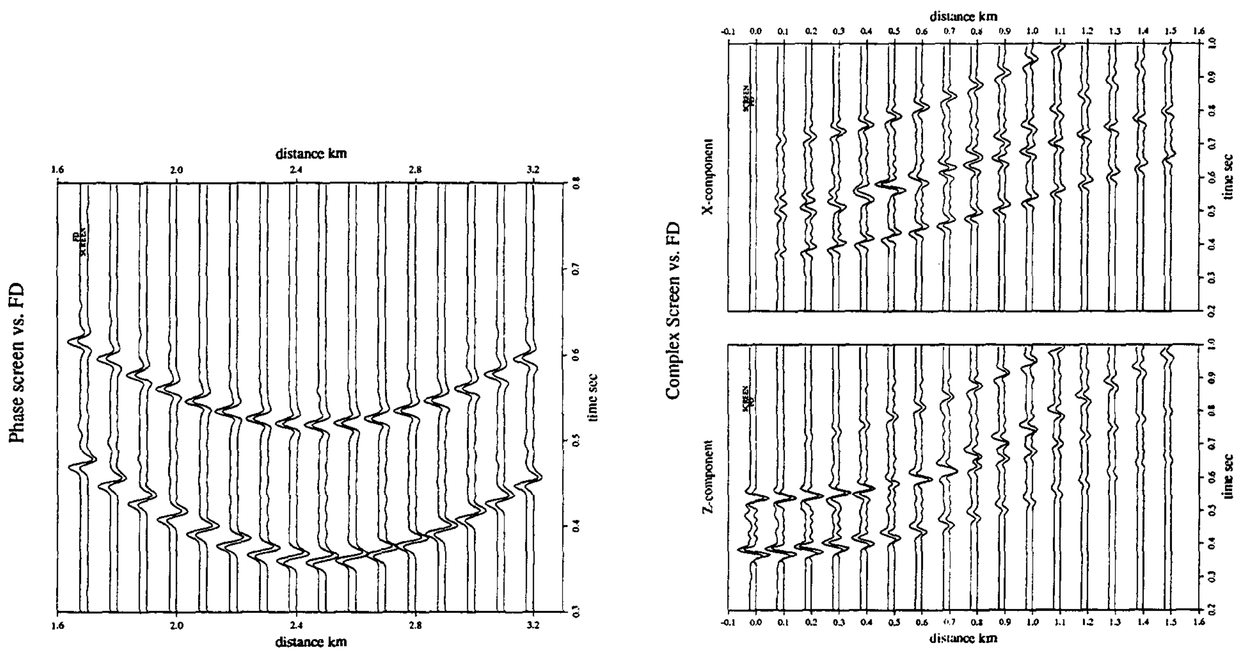


Figure 5: A model used to compare the screen approximation method with a FD method.



(a)

(b)

Figure 6: Comparison between the results of the screen method and the FD method for scalar waves (a) and elastic waves (b).